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JUL 77 G C CARTER
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Optimum Element Placement for Passive Localization.

A Paper Presented at the
1977 IEEE International Conference on
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G. Clifford/Carter
Special Projects Department

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PREFACE

This document was prepared under Project No. 720Y11, "R&D Study on Target Localization," Study Chairman, W. L. Clearwaters (Code 20), and follow on proposal effort by W. A. Von Winkle, Project No. 710Y30. This work supports Project No. A12610, "Interarray Processing (IAP)," Principal Investigator, J. B. Hall (Code 3211). The Sponsoring Activity is Naval Sea Systems Command (NAVSEA 06H2-51), Program Manager, R. Cockerill.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
This document presents both the written and oral versions of a presentation given at the 1977 IEEE International Conference on Acoustics, Speech, and Signal Processing, in Hartford, Connecticut, 10 May 1977. The presentation concerned optimum element placement for passive localization.

For mathematically tractable assumptions, arrays with a third of their limited elements at each end and the middle of a line are in some sense best

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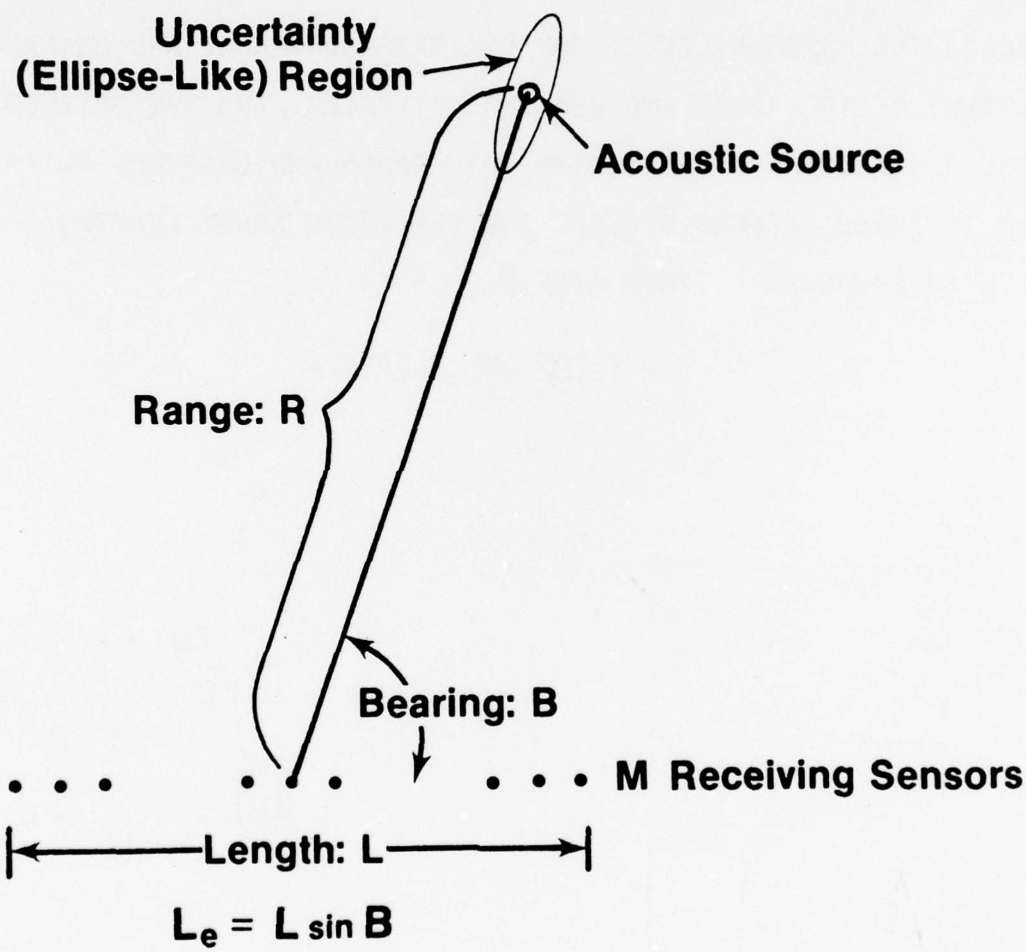
20. Cont'd:

for simultaneous range and bearing estimation. Elements in each group are spaced at half wavelengths.

INTRODUCTION

THE PURPOSE OF THIS TALK IS TO DISCUSS THE PASSIVE TECHNIQUE OF ESTIMATING THE RANGE, R, AND BEARING, B, TO AN ACOUSTIC SOURCE USING M HYDROPHONES IN A COLLINEAR ARRAY OF LENGTH, L. I WILL DISCUSS THE MAXIMUM LIKELIHOOD ESTIMATE FOR RANGE AND BEARING, THE VARIANCE OF THE RANGE AND BEARING ESTIMATES, AND THE OPTIMUM HYDROPHONE PLACEMENT FOR ESTIMATING BOTH RANGE AND BEARING. AN IMPORTANT TERM IN THESE RESULTS WILL BE THE EFFECTIVE ARRAY LENGTH, L SUB E, WHICH IS EQUAL TO L TIMES SINE B.

- SLIDE ONE PLEASE -

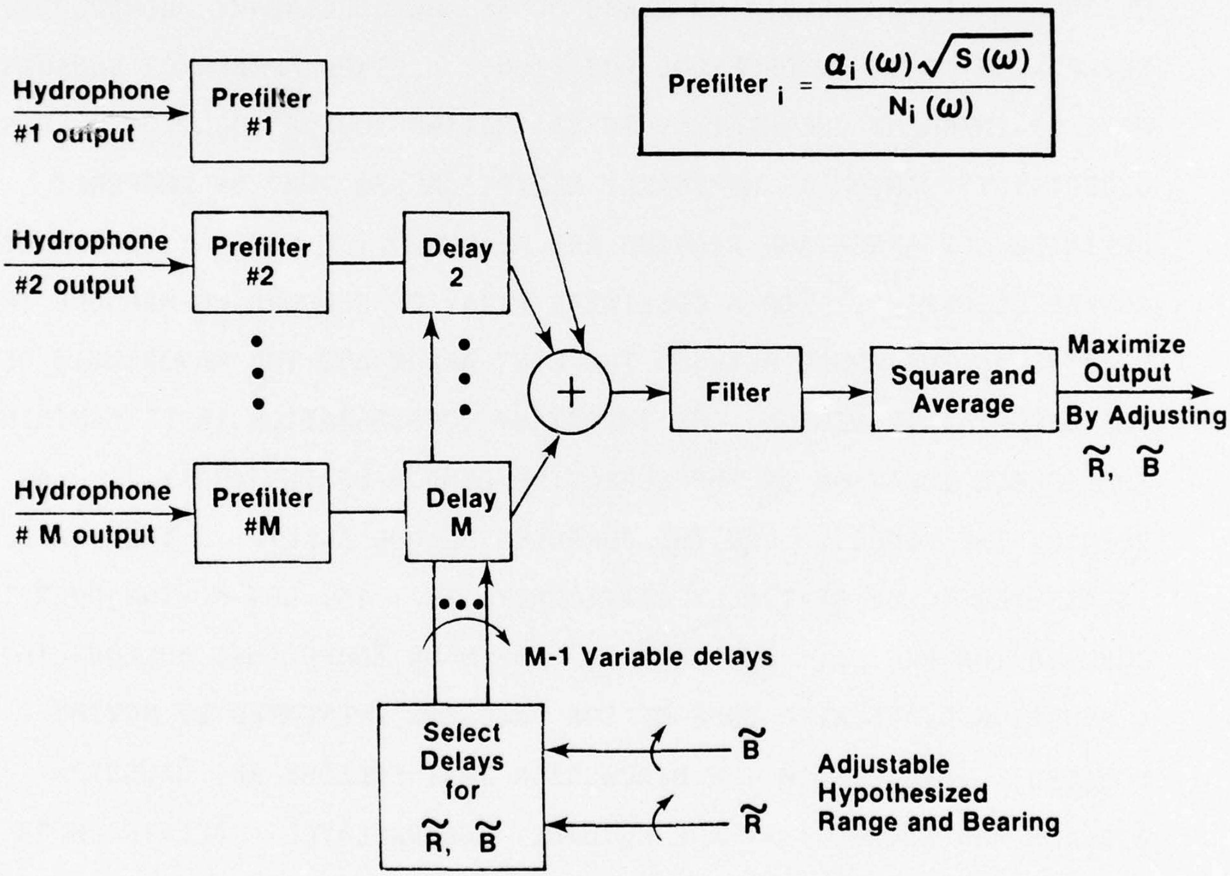

Uncertainty
(Ellipse-Like) Region
Acoustic Source
Range: R
Bearing: B
M Receiving Sensors
Length: L
$L_e = L \sin B$

SLIDE 1

SLIDE 1

THE PARTICULAR GEOMETRY OF INTEREST IS TWO DIMENSIONAL WITH AN ACOUSTIC POINT SOURCE WHOSE RANGE AND BEARING ARE TO BE ESTIMATED BY A FIXED NUMBER OF RECEIVERS. FOR THE PURPOSES OF THIS WORK, WE PRESUME THAT THE RECEIVING HYDROPHONES ARE COLLINEAR. HOWEVER, REGARDLESS OF THE HYDROPHONE POSITIONS, A FIXED NUMBER OF SENSORS HAVE AN INHERENT UNCERTAINTY IN ESTIMATING SOURCE LOCATION. THIS UNCERTAINTY REGION IS NOMINALLY ELLIPTICAL; SO THAT BY PROPERLY DEFINING HOW RANGE AND BEARING ARE MEASURED, THE ESTIMATION ERRORS CAN BE DECOUPLED. FOR A COLLINEAR ARRAY OF SENSORS, WE MEASURE THE BEARING AS THE ANGLE BETWEEN THE LINE ARRAY AND THE MAJOR AXIS OF THE UNCERTAINTY REGION. AN IMPORTANT CONSIDERATION IN DETERMINING THE SOURCE LOCATION IS THE EFFECTIVE LENGTH OF THE ARRAY, L , AS SEEN BY THE SOURCE. FOR THE PURPOSES OF OUR ANALYSIS, THE SOURCE IS ASSUMED TO BE SPATIALLY STATIONARY, THAT IS, NOT MOVING OVER THE OBSERVATION PERIOD. THE TALK BY PROFESSOR KNAPP THAT FOLLOWS THIS DISCUSSION DEALS WITH SOME OF THE PROBLEMS PRESENTED BY MOVING SOURCES. IMPLICIT IN THE DISCUSSION THAT FOLLOWS ARE GAUSSIAN SIGNALS AND NOISES THAT ARE MUTUALLY UNCORRELATED. RELATED WORK IS TREATED BY MCDONALD AND SCHULTHEISS, HAHN, AND BANGS AND SCHULTHEISS.

- NEXT SLIDE PLEASE -


Hydrophone
#1 output
Prefilter
#1
Hydrophone
#2 output
Prefilter
#2
Delay
2
Hydrophone
M output
Prefilter
#M
Delay
M
Filter
Square and
Average
Maximize
Output
By Adjusting
$\tilde{R}$, $\tilde{B}$
M-1 Variable delays
Select
Delays
for
$\tilde{R}$, $\tilde{B}$
$\tilde{B}$
$\tilde{R}$
Adjustable
Hypothesized
Range and Bearing
$\text{Prefilter}_i = \frac{\alpha_i(\omega)\sqrt{S(\omega)}}{N_i(\omega)}$

SLIDE 2

SLIDE 2

FOR A SPECIFIED ARRAY GEOMETRY, THE MAXIMUM LIKELIHOOD ESTIMATE OF RANGE AND BEARING IS OBTAINED BY COHERENTLY PROCESSING THE OUTPUTS OF THE SENSING HYDROPHONES AS SHOWN ON THIS DIAGRAM. IN PARTICULAR, EACH HYDROPHONE OUTPUT IS PREFILTERED TO ACCENTUATE HIGH SIGNAL-TO-NOISE RATIO, THEN DELAYED AND SUMMED. THE SUMMED SIGNAL IS FED TO A FILTER, THEN SQUARED AND AVERAGED FOR THE OBSERVATION TIME. THE OUTPUT OF THIS NETWORK IS MAXIMIZED THROUGH THE INDIRECT ADJUSTMENT OF THE DELAY PARAMETERS. THE DELAY PARAMETERS ARE DERIVED ON THE BASIS OF TWO ADJUSTABLE PARAMETERS HYPOTHESIZED BEARING, B TILDA, AND HYPOTHESIZED RANGE, R TILDA. THUS, AN OPERATOR NEED ONLY ADJUST HIS BEST ESTIMATE OF BEARING AND RANGE. FROM THESE TWO INPUTS, PROPER DELAYS ARE INSERTED IN EACH HYDROPHONE RECEIVING LINE. THE PROCESS OF DELAYING AND SUMMING IS A GENERALIZED BEAMFORMER WHERE THE DELAYS USED CAUSE THE BEAMFORMER TO PRESUME THE SOURCE WAVEFRONT IS CURVED AND NOT PLANAR. THE INDIVIDUAL SENSOR-TO-SENSOR DELAYS INSERTED ARE DIRECTLY RELATED TO THE HYPOTHESIZED SOURCE AND SENSOR LOCATIONS.

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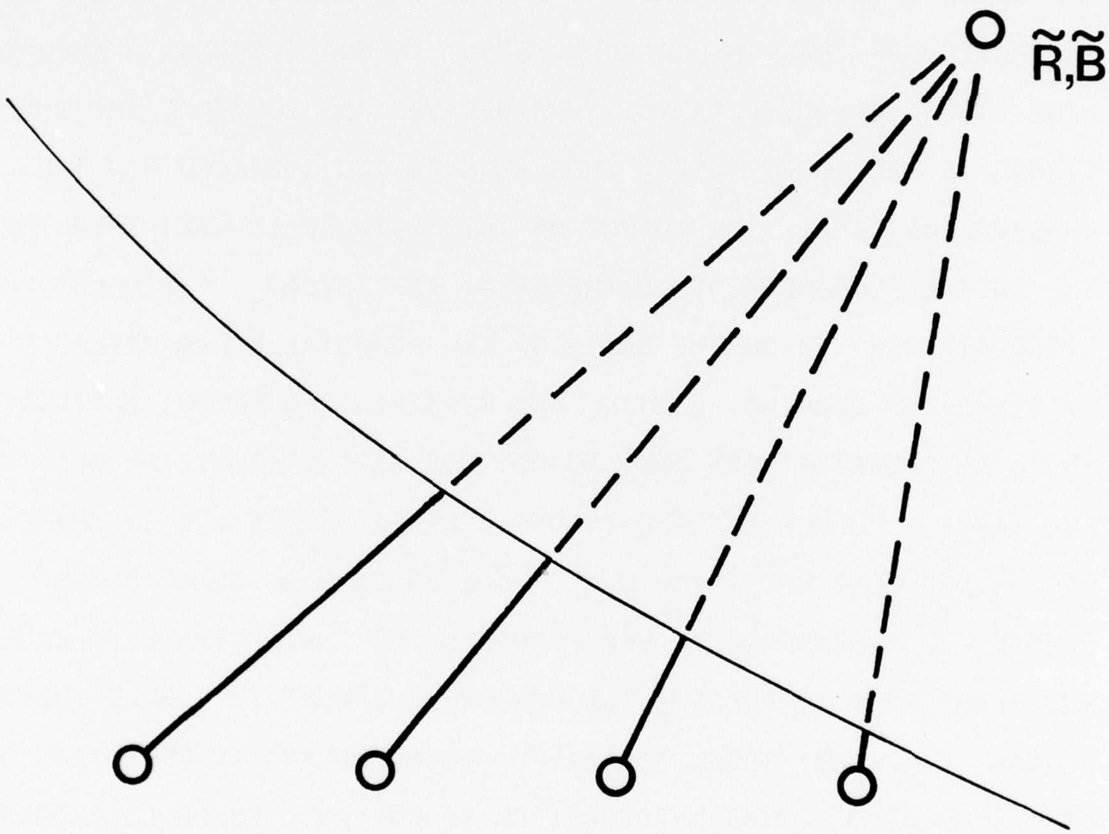

SLIDE 3

SLIDE 3

THERE ARE SEVERAL PHYSICAL INTERPRETATIONS FOR THE MAXIMUM LIKELIHOOD ESTIMATES FOR RANGE AND BEARING. ONE INTERPRETATION IS SHOWN HERE. BY HYPOTHESIZING A SOURCE POSITION CHARACTERIZED BY A FINITE RANGE, R TILDA, AND BEARING, B TILDA, A SPHERICAL OR CURVED WAVEFRONT IS PRESUMED. THIS CURVATURE AND THE ACTUAL HYDROPHONE POSITIONS GIVE RISE TO DELAYS THAT ARE INSERTED IN THE GENERALIZED BEAMFORMER TO MATCH THE PROCESSOR TO THE WAVEFRONTS CURVATURE.

- NEXT SLIDE PLEASE -

Cramér Rao Lower Bound (Min. Var.)

$$\sigma^2(\hat{\xi}) = \frac{2\pi C^2}{T\ U_\xi \int_0^\Omega V(\omega)\omega^2 d\omega}$$

Where

T is observation time

U_ξ is a function of the sensors and their positions (Different for $\xi = R$, $\xi = B$)

V is a function of the output SNR

C is the speed of sound in the medium

SLIDE 4

SLIDE 4

BANGS AND SCHULTHEISS HAVE SHOWN THAT THE CRAMÉR RAO LOWER BOUND FOR ESTIMATING A PARAMETER ξ IS CHARACTERIZED BY THE SIMPLE FUNCTION SHOWN HERE (WHEN SIGNAL-TO-NOISE RATIO AT EACH SENSOR IS THE SAME). THE PARAMETER ξ CAN DENOTE RANGE OR BEARING. IN PARTICULAR, THE VARIANCE OF THE ESTIMATED PARAMETER IS CHARACTERIZED BY TWO PI TIMES C SQUARED WHERE C IS THE SPEED OF SOUND IN WATER; OVER THE PRODUCT OF THREE TERMS. THE THREE DENOMINATOR TERMS INVOLVE T, U, AND V TO BE DISCUSSED ON THIS AND SUBSEQUENT SLIDES:

I IS THE OBSERVATION TIME.

U SUB ξ IS A FUNCTION OF THE SENSORS AND THEIR POSITIONS. FOR VARIANCE OF THE RANGE ESTIMATES WE USE U SUB R AND FOR VARIANCE OF THE BARING ESTIMATES WE USE U SUB B.

V IS A FUNCTION OF THE SIGNAL-TO-NOISE RATIO AND THE NUMBER OF SENSORS. NOTE THAT IT IS THE INTEGRAL OF V OF OMEGA OVER THE FREQUENCY BAND ZERO TO CAP OMEGA THAT IS IMPORTANT IN THE VARIANCE EXPRESSION. ALSO, HIGH FREQUENCIES ARE MORE IMPORTANT THAN LOW FREQUENCIES, DUE TO THE OMEGA SQUARED TERM IN THE INTEGRAL. THE CRAMÉR RAO BOUND PRESENTED HERE IS FOR LOCAL VARIATION AND IMPLICITLY ASSUMES HIGH OUTPUT SIGNAL-TO-NOISE RATIO.

INCREASES IN THE PRODUCT OF T, U, AND V RESULT IN DECREASES IN THE VARIANCE OF THE RANGE OR BEARING ESTIMATES. FOR THE ARRAY CONFIGURATIONS, T, U, AND V ARE INDEPENDENT; SO INCREASING T, U, OR V WILL RESULT IN DECREASED VARIANCE. FOR EXAMPLE, DOUBLING T WILL REDUCE THE STANDARD DEVIATION OF EITHER THE BEARING ESTIMATE OR RANGE ESTIMATE BY ABOUT ONE POINT FOUR. MINIMIZE THE VARIANCE BY SELECTING SENSOR LOCATIONS TO MAXIMIZE U.

- NEXT SLIDE PLEASE -

For low output SNR

$$V(\omega) \cong M^2 \frac{S^2(\omega)}{N^2(\omega)}$$

For high output SNR

$$V(\omega) \cong M \frac{S(\omega)}{N(\omega)}$$

SLIDE 5

SLIDE 5

THE DENOMINATOR TERM, V OF OMEGA, IS A COMPLICATED FUNCTION OF SIGNAL-TO-NOISE RATIO (SNR) AND THE NUMBER OF SENSORS, M. THE OUTPUT SIGNAL-TO-NOISE RATIO IS DEFINED BY THE PRODUCT OF THE INPUT SIGNAL-TO-NOISE RATIO, S OF OMEGA OVER N OF OMEGA, MULTIPLIED BY THE NUMBER OF SENSORS, M. THE FUNCTION, V OF OMEGA, BEHAVES DIFFERENTLY AT LOW AND HIGH OUTPUT SNR. FOR LOW OUTPUT SNR, THIS FUNCTION BEHAVES AS THE SQUARE OF THE OUTPUT SNR. ASSUMING WE ARE IN A PASSIVE SITUATION AND CANNOT CONTROL THE INPUT SIGNAL-TO-NOISE RATIO, WE SEE OUR ONLY METHOD FOR IMPROVEMENT IN THE V FUNCTION IS TO INCREASE M. WHEN WE ARE AT LOW OUTPUT SNR, INCREASING M HELPS DRAMATICALLY. HOWEVER, AS M IS INCREASED, THE OUTPUT SIGNAL-TO-NOISE RATIO GETS LARGER AND THE V FUNCTION IMPROVES LESS DRAMATICALLY; INDEED, INCREASING M IS LIKE INCREASING THE INTEGRATION TIME.

- NEXT SLIDE PLEASE -

Bearing Estimation

$$U_B = \frac{L_e^2}{K_B}$$

Array Type	K_B	
Equispaced Line	6	
M/2, 0, M/2	2	← Bound
M/3, M/3, M/3	3	
M/4, M/2, M/4	4	

SLIDE 6

SLIDE 6

THE TERM U THAT WE DESIRE TO MAXIMIZE IS DIFFERENT FOR RANGE AND BEARING ESTIMATION. FOR BEARING ESTIMATION WITH FOUR PROPOSED LINE ARRAYS, U SUB B IS GIVEN BY THE SQUARE OF THE EFFECTIVE ARRAY LENGTH DIVIDED BY A CONSTANT. THE EFFECTIVE ARRAY LENGTH IS EQUAL TO THE PHYSICAL ARRAY LENGTH TIMES THE SINE OF THE BEARING TO THE SOURCE. THUS, IT IS THE APPARENT LENGTH OF THE ARRAY AS SEEN BY THE SOURCE. IN BEARING ESTIMATION, WE DESIRE TO MAKE L SUB E LARGE AND THE CONSTANT K SUB B SMALL IN ORDER TO REDUCE VARIANCE. NOTE THAT DOUBLING THE ARRAY LENGTH REDUCES THE VARIANCE BY FOUR. THUS, ARRAY LENGTH IS A MORE IMPORTANT FACTOR IN BEARING ESTIMATION THAN EITHER INTEGRATION TIME OR THE NUMBER OF HYDROPHONES WHEN ONE IS ALREADY AT HIGH OUTPUT SIGNAL-TO-NOISE RATIO.

THE FOUR DIFFERENT ARRAY TYPES STUDIED ARE AN EQUISPACED LINE ARRAY AND THREE ARRAYS WITH M ELEMENTS GROUPED AT THE TWO ENDS AND THE MIDDLE OF THE ARRAY. MCDONALD AND SCHULTHEISS HAVE SHOWN THAT BY PLACING HALF OF THE M ELEMENTS AT EACH END OF A LINE ARRAY IN AN M OVER TWO, ZERO, M OVER TWO GROUPING, A BOUND ON BEARING VARIANCE IS OBTAINED. THREE GROUPS OF A THIRD EACH RESULT IN A K SUB B OF THREE AND GROUPINGS OF QUARTER, HALF, QUARTER RESULT IN A K SUB B OF FOUR.

THE PRACTICAL IMPLICATIONS OF MCDONALD AND SCHULTHEISS'S RESULT ARE BOTH TO PROVIDE A BOUND ON HOW WELL BEARING COULD BE ESTIMATED UNDER IDEAL CONDITIONS AND TO SUGGEST HOW TO PLACE A LIMITED NUMBER OF HYDROPHONES OVER A LARGE APERTURE. NOTABLY HALF OF THE HYDROPHONES SHOULD BE POSITIONED AT EACH END OF THE ARRAY, PLACED AT HALF-WAVE LENGTH SPACING FOR THE DESIGN FREQUENCY.

-NEXT SLIDE PLEASE-

Range Estimation

$$U_R = \frac{L_e^4}{K_R R^4}$$

Array Type	K_R	
Equispaced Line	360	
M/2, 0, M/2	∞	
M/3, M/3, M/3	144	
M/4, M/2, M/4	128	← Bound

SLIDE 7

SLIDE 7

THE TERM U FOR RANGING DEPENDS ON THE FOURTH POWER OF THE RANGE RELATIVE TO THE EFFECTIVE BASELINE. THE VARIANCE OF THE RANGE ESTIMATE IS REDUCED BY MAKING THE EFFECTIVE ARRAY LENGTH L SUB E LARGE. THIS CAN BE DONE BY MAKING THE ARRAY LENGTH L LARGE OR BY PHYSICALLY STEERING THE ARRAY BROADSIDE TO THE SOURCE. THE VARIANCE CAN ALSO BE REDUCED BY DECREASING THE RANGE TO THE SOURCE. MOST NOTABLE IN THE U SUB R TERM IS THAT THE VARIANCE OF THE RANGE ESTIMATE INCREASES AS THE FOURTH POWER OF THE RATIO OF RANGE TO THE EFFECTIVE ARRAY LENGTH. THIS FOURTH POWER DEPENDENCE OF THE VARIANCE ON ARRAY LENGTH SUGGESTS THE NEED TO MAKE L SUB E LARGE IN ORDER TO ESTIMATE RANGE.

THE CONSTANT K SUB R DEPENDS ON THE ARRAY TYPE. FOR AN EQUISPACED LINE ARRAY, K SUB R IS 360. A BOUND IS PROVIDED BY AN ARRAY CONFIGURED WITH A QUARTER OF THE HYDROPHONES AT EACH END AND HALF IN THE MIDDLE. THUS, WE SEE THAT THE ARRAY CONFIGURATION DESIRED FOR BEARING ESTIMATION AND THE ONE FOR RANGE ESTIMATION DIFFER. THE BEARING ARRAY SHOULD HAVE ITS ELEMENTS TOWARD THE ARRAY ENDS, WHILE THE RANGING ARRAY SHOULD HAVE HALF OF ITS ELEMENTS IN THE CENTRAL PORTION.

AS POINTED OUT IN THE CONFERENCE PROCEEDINGS, AN ARRAY WITH A THIRD OF ITS ELEMENTS AT EACH END AND THE MIDDLE WILL MINIMIZE THE UNCERTAINTY REGION. THUS, IN SOME SENSE AN ARRAY COMPRISED OF THREE EQUAL GROUPS OF ELEMENTS WILL OUTPERFORM ALL OTHER ARRAYS FOR PASSIVELY LOCATING AN ACOUSTIC SOURCE.

- NEXT SLIDE PLEASE -

Relative Range Errors

$$\frac{\sigma(\hat{R})}{R} = \sqrt{\frac{K_R}{K_B}} \left[\frac{R}{L_e}\right] \sigma(\hat{B})$$

Array Type	$\sqrt{\frac{K_R}{K_B}}$
Equispaced Line	7.75
M/2, 0, M/2	∞
M/3, M/3, M/3	6.9
M/4, M/2, M/4	5.7

SLIDE 8

SLIDE 8

THE RELATIVE RANGE ERROR GIVEN BY THE STANDARD DEVIATION OF THE RANGE ESTIMATE DIVIDED BY THE TRUE RANGE IS GIVEN BY A CONSTANT TIMES THE STANDARD DEVIATION OF THE BEARING ESTIMATE (MEASURED IN RADIANS) TIMES A TERM THAT DEPENDS LINEARLY ON THE RANGE TO THE SOURCE RELATIVE TO THE EFFECTIVE ARRAY LENGTH. IT IS IMPORTANT TO NOTE THAT THE RELATIVE ERROR INCREASES LINEARLY WITH ACTUAL RANGE TO THE SOURCE. STANDARD DEVIATIONS OF RANGE ESTIMATES INCREASE AS THE SQUARE OF THE RANGE TO THE SOURCE. HENCE WE SEE THAT IT IS EXTREMELY DIFFICULT TO ESTIMATE RANGE OF A DISTANT SOURCE EVEN UNDER IDEAL CONDITIONS.

ONE OF THE ADVANTAGES OF EXPRESSING RELATIVE RANGE ERRORS IN THIS FORM IS THAT THE STANDARD DEVIATION OF BEARING ESTIMATES IS A TERM FAMILIAR TO SONAR ENGINEERS AND SIGNAL PROCESSORS. MOREOVER, THERE IS CONCERN THAT THE OCEAN MEDIUM MAY INHERENTLY LIMIT THE PRACTICAL ABILITY TO ESTIMATE BEARING EVEN THOUGH THEORY PREDICTS WITH ENOUGH SIGNAL-TO-NOISE RATIO OR INTEGRATION TIME, THE BEARING CAN BE MEASURED ARBITRARILY WELL. THE EXPRESSION GIVEN HERE CLEARLY POINTS OUT THE NEED TO MAKE THE ARRAY LENGTH LARGE WHEN THE SOURCE RANGE CANNOT BE REDUCED. OF INTEREST IS THAT THIS CONCLUSION IS EXTREMELY INSENSITIVE TO THE TYPE OF ARRAY, PROVIDED THE ARRAY HAS SOME RANGING CAPABILITY. THIS CAN BE SEEN FROM THE SIMILARITY OF THE CONSTANTS GIVEN.

- NEXT SLIDE PLEASE -

FOR ESTIMATING	BEST ARRAY CONFIGURATION		
BEARING	● ● ● ● ● ● M/2		● ● ● ● ● ● M/2
RANGE	● ● ● M/4	● ● ● ● ● ● M/2	● ● ● M/4
POSITION	● ● ● ● M/3	● ● ● ● M/3	● ● ● ● M/3

SLIDE 9

SLIDE 9

TO SUMMARIZE, WE DESIRE TO KNOW HOW TO PLACE A LIMITED NUMBER OF HYDROPHONES OVER A SPECIFIED BASELINE. THE HYDROPHONES SHOULD BE PLACED IN GROUPS, WITH THE HYDROPHONES IN EACH GROUP PLACED AT HALF WAVE LENGTH SPACING FOR THE DESIGN FREQUENCY. FOR BEARING ESTIMATION, HALF OF THE M HYDROPHONES SHOULD BE PLACED AT EACH END OF THE ARRAY. FOR RANGE ESTIMATION, A QUARTER OF THE HYDROPHONES SHOULD BE PLACED AT EACH END OF THE ARRAY AND HALF PLACED IN THE MIDDLE. FOR SIMULTANEOUSLY ESTIMATING RANGE AND BEARING, THE HYDROPHONES ARE PLACED IN THREE GROUPS EACH WITH M OVER THREE HYDROPHONES.

SLIDE OFF PLEASE. ARE THERE ANY QUESTIONS?

METHODS FOR PASSIVELY LOCATING AN ACOUSTIC SOURCE

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ABSTRACT

The maximum likelihood (ML) processor is presented for passively estimating range and bearing to an acoustic source. The source signal is observed for a finite time duration at several sensors in the presence of uncorrelated noise When the speed of sound in an isovelocity medium and the sensor positions are known the ML estimator for position constrains the source to sensor delays to be focused into a point corresponding to a hypothesized source location. The variances of the range error and bearing error are presented for the optimum processor. It is shown that for bearing and range estimation different sensor configurations are desirable. However, if the area of uncertainty is to be minimized, then the sensors should be divided into equal groups with one third of the sensors in each group.

INTRODUCTION

An (underwater) acoustic point source radiating energy to four (or more) receiving sensors is shown in Figure (1). The position of the source (in two space) can be characterized by polar coordinates (range and bearing) for a given frame of reference.

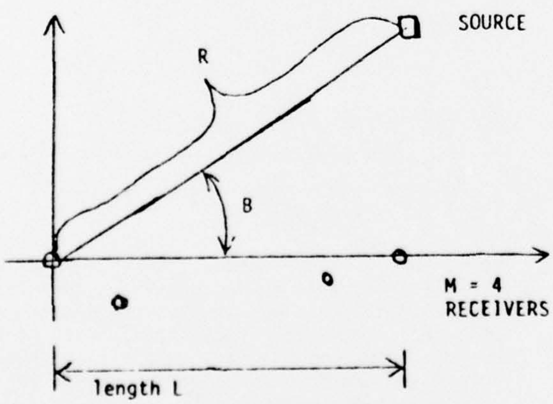

FIGURE (1) R, B, L, M Defined

For example, the range, R, and bearing, B, relative to a specified sensor and line from the sensor can be used as shown in Figure (1). The problem we address here is how to estimate range, R, and bearing, B, to a source when M sensors separated by a maximum of L(meters) have observed T seconds of received data. We will examine the maximum likelihood (ML) technique for position estimation.

MATHEMATICAL MODEL

For our purposes we assume that each receiving sensor at the t-th instant in time corresponds to a signal plus noise. Namely, the i-th (of M) sensor outputs is characterized by

$$r_i(t) = s_i(t) + n_i(t), \; i = 1, M \quad (1)$$
$$0 \le t \le T.$$

The signal and noise are uncorrelated and the noises are mutually uncorrelated.

For a spatially stationary source the signal can be viewed as a (range and frequency dependent attenuated and) delayed source signal. More generally, source motion will impart a time compression (or expansion) to the received signal wave form. Source motion is discussed by Knapp, Carter and others, in this conference, but is beyond the scope of this paper. However, we know the problem of estimating the position of a stationary source is considerably easier than that of a moving source. Thus, the results in this paper serve as a bound on performance (variance); still we will see that it is extremely difficult in the best case to estimate source range. We know further (for a moving source) if the time compression varies (significantly) across the sensor baseline that failure to compensate for source motion will (significantly) degrade performance. Extracting estimates of time compression may aid in motion analysis and indeed may be necessary but certainly performance would be better against a stationary source (or moving source with known motion).

While in general, attenuation is a function of frequency and range, we can initially treat attenuation as a constant and later impose the frequency (and range) dependence. In particular, we can model the i-th signal as

$$s_i(t) = \alpha_i s(t + D_i), \; i = 1, M \quad (2)$$

where s(t) is the signal at the source.

(Without loss of generality, we can assume $\alpha_i = 1$ and $D_1 = 0$.)

There has been considerable attention devoted to estimating the (relative) time delay parameter vector $(D_2 - D_1, D_3 - D_1, \ldots, D_M - D_1)$. See for example, MacDonald and Schultheiss (1969), Bangs and Schultheiss (1973), Hannan and Thompson (1973),

Hahn (1975), and Knapp and Carter (1976).

BEARING ESTIMATION

The relative delay between two receiver (or beamformer) outputs is related to the bearing of the source. In particular, the bearing of the source relative to the i - th and (i - 1)-st sensor is

$$B_{i,\, i-1} = \arc\cos\left[\frac{C(D_i - D_{i-1})}{d}\right] \quad (3)$$

where C is the speed of sound in a nondispersive isovelocity medium and d is the separation between the two sensors (or beamformer centers). The bearing relative to the defined reference line is known when the bearing to the source relative to the i - th and (i - 1)st sensor is known and the position of those sensors is known (relative to the reference line). In particular, the effective bearing angle is what you would see at two sensors corrected by how the two sensors are out of alignment (relative to the reference line). In practice, errors in bearing are caused by estimation errors and by uncertainities in knowing the sensor location. If the sensor positions vary with time about some known position, time averaging will correct for position variation. If the sensor positions vary about some unknown position, significant biases in bearing (range) estimation can exist that cannot be averaged out. The estimation errors are a function of integration time, processing bandwidth and coherence (or signal-to-noise ratio (SNR)) between the two sensors, as well as inhomogenities in the medium causing wave front warping (difference in C between the source and various sensors). It can be shown that (for many practical problems) we can transform our problem into an equivalent one in which the range and bearing to a source are to be measured by measuring the effective bearings to a linear array. Thus, we concentrate our attention on linear arrays. When the effective bearings are known exactly, four or more sensor bearing (pairs) provide a redundant characterization of the source position. If, however, we estimate the bearings and plot the bearing lines (or asymptotes to hyperbolic curves) then, in general, the lines will not intersect in a single point.

From the discussion of the mathematical model, we can see a method evolving to estimate the position of the source. This method focuses all bearing lines into one hypothesized source position.

MAXIMUM LIKELIHOOD

The maximum likelihood (ML) estimate for the time delay vector has been derived for stationary Gaussian processes by Hahn (1975) and Carter (1976). As stated earlier, these time delay estimates give rise to several (M-1) bearing lines (from (3)) which do not necessarily intersect in one point. It is conceivable that a point that is the (weighted) least squares distance to all bearing lines is not the best (maximum likelihood) estimate of source position. It is not difficult to show that ML estimate for range and bearing when the sensor (element) positions are known is achieved by a simple variation on Hahn (1975) and Carter (1976). In particular, by focusing all the time delay elements at many (hypothesized) range and bearing pairs and watching for the peak output of the ML time delay vector system, the ML position estimate is observed. Another way to look at this problem is that we want to maximize a quantity by adjusting a number of delay parameters subject to the constraint that all the delays must focus (intersect) in a (nominally) single (hypothesized) position. Equivalently, rather than adjust M-1 delay dials and then have to figure out a least squares position from them, we can adjust one range and one bearing dial, and by a simple (perhaps precomputed) trigonometric mapping, select all (M-1) hypothesized delays necessary to focus the sensors at one point in two space. The performance of such a system has been examined by Bangs and Schultheiss (1973).

For constant attenuation, when (and only when) we utilized the (proper frequency dependent) prefilters given by (see Bangs and Schultheiss (1973)), Hahn (1975), Carter (1976), and Knapp and Carter (1976).

$$h_k \doteq \frac{S(\omega_k)/N^2(\omega_k)}{1 + M\,\dfrac{S(\omega_k)}{N(\omega_k)}} \quad (4)$$

(where S and N denote signal and noise power spectral densities), then the variance of the parameter θ (where $\theta = \hat{R}$ or $\theta = \hat{B}$) is

$$\sigma^2(\theta) = 2\pi\left[E\,\mathrm{Tr}(P_\theta P_\theta^*)\,T\Delta\omega\,\Sigma\, S(\omega_k)\, h_k \omega_k^2\right]^{-1} \quad (5)$$

where * denotes complex conjugate transpose, Tr denotes the trace, P_θ is a particular position matrix, and E is an effective degradation factor both to be defined. For bearing estimation, the i - th, j - th element of the P_B matrix is given by

$$P_{ij}^{(B)} = \frac{\sin B}{C}\,(Z_i - Z_j) \quad (6)$$

where Z_n designates the position of the n - th sensor. For range estimation, the i - th, j - th element of the P_R matrix is given by

$$P_{ij}^{(R)} = \frac{-\sin^2 B}{2CR^2}\,(Z_i^2 - Z_j^2) \quad (7)$$

The degradation factor is defined by

$$E = \frac{\mathrm{Tr}(P_B P_B^*)\,\mathrm{Tr}(P_R P_R^*) - \left[\mathrm{Tr}(P_B P_R^*)\right]^2}{\mathrm{Tr}(P_B P_B^*)\,\mathrm{Tr}(P_R P_R^*)} \quad (8)$$

Thus, when (but only when) $\mathrm{Tr}(P_B P_R^*) = 0$ we see that E = 1 and there is no degradation. However, in general, this will not be the case and E must be computed. Proper (or fortuitous) choice of the coordinate system will insure E = 1 and then only $\mathrm{Tr}(P_\theta P_\theta^*)$ plays a role in the expression for variance of the estimates.*

An important term in the variance expression is

$$S(\omega_k) h_k = \frac{a^2}{1 + Ma} \quad (9)$$

where $a = S(\omega_k)/N(\omega_k)$

At first glance, it may appear that the variance

*Errata: We can show for a symmetric line array (SLA), with the origin in the center, that E=1. All the results that follow apply to bearings and ranges measured to the center of an SLA. A more complete discussion will be given in a subsequent paper.

could be reduced by increasing h; however, if the sensor positions are fixed then the variance, given by (5), is the minimum variance obtainable and is attained when h_k is of the form specified by (4). For low output SNR (Ma < < 1) (9) goes as the square of the SNR. For high output SNR (9) goes as the ratio of SNR to M.

When the number of sensors and the SNR fix the output SNR, the performance (variance) is governed only by $ETTr(P_\theta P_\theta^*)$. Increasing the observation time, T, decreases the variance according to (5). We desire to know how to decrease the variance through proper element placement. MacDonald and Schultheiss (1969) have stated that for bearing estimation half of the elements should be placed at each end of an array of fixed length (provided the noise remains uncorrelated). In practice, if we are constrained to have M elements in an array of length L, we should place M/2 by beginning at each end and spacing every half wave length (for the design frequency) towards the center.

For an equispaced line array (d spacing) we find for large M and L = (M-1)d

$$\left[ETr(P_B P_B^*)\right]^{-1} = \frac{6C^2}{L_e^2 M^2} \qquad (10)$$

where the effective array length projection seen by the source is given by Le = LsinB, which agrees with eq(34) of Bangs and Schultheiss (1973) and agrees with eq(19) of MacDonald and Schultheiss (1969). As expected, good performance (low variance) is obtained by steering broadside to the source and making the array length, L, and number of sensors, M, as large as possible. In theory, the variance can be reduced by a factor of three by proper element placement (That is, the 6 in (10) is replaced by a 2).

The ranging performance, for an equispaced line array can be shown to be governed by

$$\left[ETr(P_R P_R^*)\right]^{-1} = \frac{360\, C^2 R^4}{L_e^4 M^2} \qquad (11)^*$$

This result agrees with Bangs and Schultheiss (1973). In expression (11) we see the tremendous importance of absolute range to the source (variance goes as a fourth power).* In order to corroborate the derivation, $ETr(P_R P_R)$ was numerically computed and the equispaced results agreed with the derivation as expected. Nuttall (1976) has shown that if the array elements were symmetrically placed about some origin that the best ranging performance (lowest variance) could be achieved by putting one half the elements in the middle and one quarter at each end of the array. (However, there can be non-symmetric ranging arrays that outperform the symmetric array.) For an array of quarter, half, quarter grouping it can be shown that

$$\left[ETr(P_R P_R^*)\right]^{-1} = \frac{128\, C^2 R^4}{L_e^4 M^2} \qquad (12)^*$$

which outperforms the equispaced array by a factor of 2.8 in variance, and is conjectured to be the best ranging array geometry for large M.

There is a difference between how to place M elements of a linear array depending on whether range or bearing is to be estimated). If we define that the average error vector be minimized between our position estimate and the true source position then it can be shown that the sensor positions will be a function of the true range to the source. (In particular, for close-in sources we will want a (M/2,0, M/2) division of sensors and for very distant sources we will want element groups of M/4, M/2, M/4.) One way of defining a best array geometry is to minimize the area of uncertainty. That is, select the sensor positions to minimize the quantity $R\sigma(\hat{B})\sigma(\hat{R})$. Such an area of uncertainty will clearly not be square; for example, at large ranges we will be more apt to make range errors than bearing errors. The optimum placement was found numerically to be M/3, M/3, M/3. For the one third, one third, one third division

$$\left[ETr(P_B P_B^*)\right]^{-1} = \frac{3C^2}{M^2 L_e^2} \qquad (13)$$

and

$$\left[ETr(P_R P_R^*)\right]^{-1} = \frac{144\, C^2 R^4}{M^2 L_e^4} \qquad (14)^*$$

Thus, by placing M/3 sensor elements at -L/2, M/3 elements at 0 and M/3 elements at L/2 we achieve a comprise bearing and range estimation system; such a comprise minimizes the area of uncertainty defined by $R\sigma(\hat{R})\sigma(\hat{B})$. Attempts to minimize other error measures can result in array configurations that depend on source range.

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*Errata: The coefficients, 360, 128 and 144, in equations (11), (12) and (14) given here are believed to be correct; they are 16 times larger than reported in the original manuscript.

ERRATA AND APPENDIX

METHODS FOR PASSIVELY LOCATING AN ACOUSTIC SOURCE

Dr. G. Clifford Carter

1. Comparing eq. (4) and (9) we see an inconsistency. Equation (4) should be

$$h_k = \frac{S(\omega_k)/N^2(\omega_k)}{1 + M\,\frac{S(\omega_k)}{N(\omega_k)}} ,$$

where h_k is the magnitude squared transfer function.

2. If in eq. (5) we define

$$\Delta\omega = \frac{2\pi}{T}$$

as is usually done, then equation (5) must be multiplied by 2π in order to be correct. The rationale for the form of (5), which is modified from Bangs and Schultheiss (1973), is, of course, to: (1) show the dependence on integration time and (2) allow the summation to be replaced by integration. It should also be pointed out that eq. (5): holds only for high output SNR, involves the local variation of the parameter estimate and, ignores ambiguous parameter estimates due, for example, to large sidelobes.

3. Following the discussion (about E=1 near eq. (9))) a comment should have been inserted that "we can show for a symmetric line array (SLA), with the origin in the center, that E=1. All the results that follow apply to bearing and ranges measured to the center of an SLA and not to Figure 1 of the text. A more complete discussion will be given in a subsequent paper."

4. The coefficients in equations (11), (12) and (14) should be 360, 128 and 144 respectively; that is, the ranging variance is 16 times worse than quoted in the uncorrected manuscript.

5. The degradation factor, eq. (8), can be viewed as follows. For any array and any source a fixed ellipse of uncertainty exists for the particular fixed SNR, M, T, L, B, and R. The absolute position and area of this ellipse do not change as a function of the coordinate system. However, if when we select the coordinate system we define the range and bearing as being measured from the center of the coordinate system then the variance of the range and bearing estimates do change as a function of the coordinate system.

6. The minimum area of uncertainty is not necessarily obtainable. Thus equations (13) and (14) serve as a bound on performance, and further, do not consider issues of beam pattern sidelobes causing ambiguous estimates.

In practice, if we had a limited number of elements, they should be placed at half wavelength spacing (for a given design frequency) in three groups, with M/3 at each end of the baseline and M/3 in the center. It is useful to note for three groups of M/3 that theoretically

$$\frac{\sigma(\hat{R})}{R} = 4\sqrt{3}\,\frac{R}{L_e}\,\sigma(\hat{B})$$

where $\sigma(\hat{B})$ is measured in radians. Increases in M, T or SNR decrease $\sigma(\hat{B})$ and thus only indirectly decrease the relative range error. Increases in L_e effect both $\sigma(\hat{B})$ and the relative range error. It is also instructive to look at the ratio of the major and minor axes of the uncertainty ellipse. We note

$$\frac{\sigma(\hat{R})}{\sigma(\hat{B})} = 4\sqrt{3}\,\frac{R^2}{L_e} .$$

Thus, the shape of the uncertainty ellipse changes depending on range and effective baseline.

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